Band pass filter design
Part 1. Band pass filters from first principles

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1. Introduction

Filters are central to building radio equipment with good performance and have long been one of my interests. Good front end filtering of a receiver reduces inter-modulation and a well-designed CW/SSB filter will enable signals to be read with high levels of adjacent channel interference. Interference caused by harmonics of the carrier of a transmitter can be eliminated by means of a band pass filter. (I am, of course, referring to analogue filters – I will leave digital filters to others…)

I will admit to building filters in transmitters and receivers in the past, based on a few back-of-the-envelope calculations. Several years ago, I became interested in designing (as opposed to cobbling together or copying) crystal IF filters for transmitters and receivers, with a long term aim of making a moderate performance transceiver. The first few months were spent designing and building test equipment to measure the motional parameters (the equivalent circuit) of individual quartz crystals and the overall performance of prototype filters that I had designed. Three years later I am still working on the design of band pass LC and crystal filters, although in the meantime I have learnt a great deal about the design of filters generally and band pass filters in particular.

Although software is available to assist with the design of filters (for example Refs 10 and 11), using it does not yield many insights into filter design. So, as a separate project, I set myself the target of designing band pass and crystal filters from first principles. This series of articles outline some of the techniques that I used.

Part 1 examines the steps necessary to go from standard tables of component values for low pass filters to a completed design of a band pass filter using three top coupled tuned circuits. This Part introduces the concepts of impedance inverters and negative value capacitors. Impedance matching is also considered.

Part 2 describes the alternative (and easier) “q & k” method of band pass filter design

Part 3 introduces the use of Norton’s Transform to change impedance levels within a band pass filter

Part 4 describes the use of Star-to-Delta transformations to improve the frequency response in applications where harmonic suppression is important

Part 5 looks at Bartlett’s Bi-section Theorem, which is an alternative means of changing impedance levels within filters with circuit diagrams that are symmetrical about the centre of the filter.

Part 6 will examine the effects of losses in the inductors.

Part 7 looks at impedance matching.

Part 8 examines how the degradation in filter response caused by inductor losses can be corrected.

Part 9 gives details of measurements on a practical filter

Further parts may follow.

Filter design requires compromises and the designer may not have complete control over the choice of parameters. Inductors are usually key components and designing a filter to use either standard value inductors or home-wound inductors may determine other parameters, such as terminating resistance. Capacitors are usually much less of a problem and can often be made up from low-cost 5% NPO ceramic capacitors costing pennies or more expensive 1% silver mica capacitors. A good LC meter, such as that sold by AADE (Ref. 10), is very useful, perhaps even essential, particularly if you wind your own inductors.

2. Tables of normalised filters

The starting point is the low-pass filter. There are number of well-known frequency responses for low-pass filters, such as Butterworth, Chebychev, Elliptical (or Cauer), Gaussian and Legendre. Each has advantages and disadvantages and represents various compromises between the pass-band ripple, stop band attenuation and phase response.

The two versions of a third order (three reactive components) low pass filter are shown in Fig. 1a and Fig. 1b

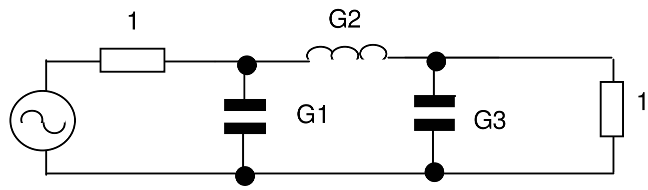

Fig 1a. Third order low pass filter - capacitive input

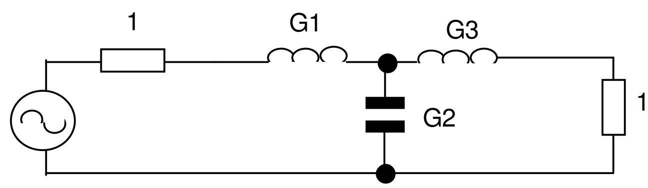

Fig 1b. Third order low pass filter - inductive input

The characteristics of these filters are defined by the transfer function, the mathematical relationship between the input and the output voltages. From the transfer function, tables of component values have been calculated and appear in many publications (Refs 1 to 4, 7 and 8). These are usually normalised to a cut-off frequency of 1 radian/sec ($1/(2\pi)$ Hz or 0.1592Hz) and a terminating resistance of 1 Ohm. If you haven't met "radians" before, these are "nature's angles" and there are 2π radians in 360 degrees – so 1 radian = 57.3 degrees. For most filters the terminating resistance will be the same as the source resistance unless a matching section is used and many filters will be symmetrical about the middle component.

Table 1 shows the component values for several third order filters, as shown in Fig 1, normalised to 1 Ohm and 1 radian/sec. One advantage of normalisation to 1 Ohm and 1 radian/sec is that the values given in the table can be either inductances (in Henries) or capacitances (in Farads), depending on whether the end components of the filter are shunt capacitors as in Fig 1a or series inductors as in Fig 1b.

Response	G1	G2	G3
Butterworth	1.0000	2.0000	1.0000
Chebychev with 0.1dB ripple	1.0316	1.1474	1.0316
Chebychev with 1.0dB ripple	2.0237	0.9941	2.0237
Gaussian	2.196	0.9674	0.3364

Table 1. Values for three section low pass filter normalised to 1 Ohm & 1 rad/sec

Most filters designed for amateur radio applications are band pass filters, for example front-end filtering of a receiver, inter-stage coupling of a transceiver and SSB/CW IF filters. We will go through the steps necessary to design a band-pass filter, suitable for the front end of a receiver, starting with the normalised values in Table 1. Software is available to design such filters but will not give an understanding of the steps in the design. If these steps are understood, then other filter topologies can be devised to give a better performance in specific applications.

For a band pass filter the following decisions have to be made:

- Filter type e.g. Butterworth, Chebychev, etc.
- Order of filter (i.e. number of inductors and capacitors in low pass prototype)
- Centre frequency
- Bandwidth
- Terminating resistance

The effects of losses in components, mainly the inductors, will be investigated separately by using simulation software. Your writer uses a free evaluation SPICE-based program from SIMetrix (Ref 9). This application has full functionality but caters for a limited number of components (in the region of 100) but this is more than enough for the simulation and the comparison of filters.

The responses displayed will be the *insertion loss* of the filter.

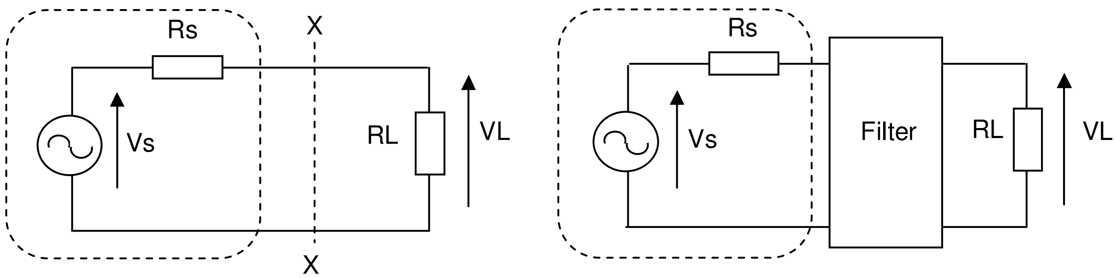

Fig. 2a. Generator and Load **Fig. 2b. Generator, filter and load**

Fig. 2a shows a signal generator, consisting of an alternating source with an output resistance driving a load. Typically, for RF work the output resistance will be 50 ohms. Maximum power transfer occurs when the load resistance RL equals the source resistance Rs. If RL = Rs then VL is half Vs – in other words there is a loss of 6dB. If we now put a filter between the source and the load at position X-X, then any additional loss is due to the *insertion loss* of the filter. If, at a particular frequency, the filter does not add any additional attenuation, then the insertion loss at that frequency is 0dB. Many instruments used for RF work have input impedances of 50 Ohms.

3. A practical design

For this exercise, we will design a band pass filter with the following characteristics:

- Chebychev with 1dB pass band ripple (so that the ripple is visible on simulations)
- Third order, capacitor input
- Centre frequency (Fo) of 10MHz
- Bandwidth (F_{BW}) of 500kHz
- 1 k-Ohm termination (R_T)

We may need to revise one or more of the items of the above specification so that inductors have specific values e.g. standard values, such as 1uH, or the inductance of home wound toroids. For many filters, the bandwidth is defined as the frequency at which the response has dropped by 3dB. In the case of the Chebychev filter, the bandwidth is the frequency at which the response has dropped to value equal to the pass band ripple (1dB in this example). Correction factors can be calculated for the frequency at which the attenuation is 3dB and some designs use the 3dB frequency with this correction.

The first step is to take the normalised filter component values, derived from the standard tables, and change the component values to make a 500kHz low pass filter terminated in 1k-Ohm. There are two possible configurations, capacitor input or inductor input. For this exercise we will use the capacitor input version, as shown in Fig. 1a. The inductors and capacitors are scaled as follows:

$$L_{Filter} = Ln\ R_T / (2\pi\ F_{BW}) \quad (1)$$

$$C_{Filter} = Cn / (2\pi\ F_{BW}\ R_T) \quad (2)$$

where Ln and Cn are the normalised component values from the Table 1, R_T is the terminating resistance. Using the circuit drawing facility of the SIMetrix SPICE simulation program, the circuit diagram and simulated insertion loss is shown in Fig 3 and 4 respectively.

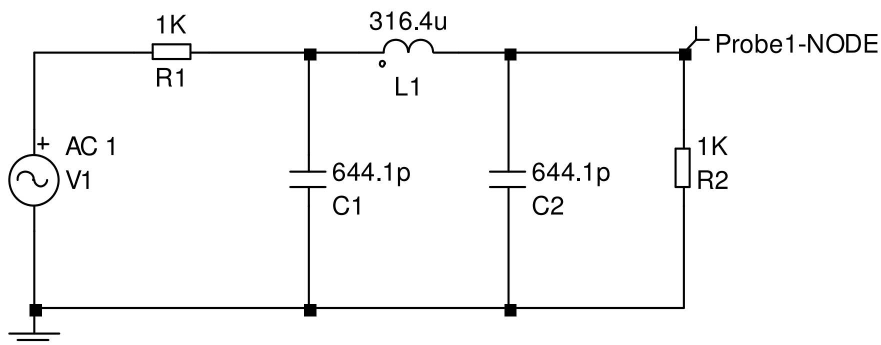

Fig. 3. Chebychev 500kHz low pass filter with 1dB pass band ripple

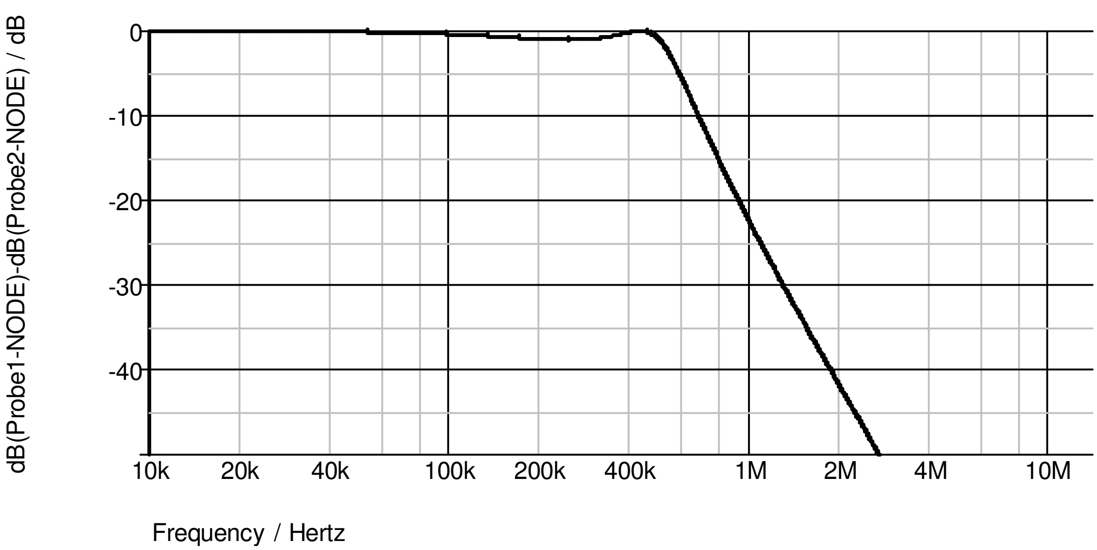

Fig 4. Insertion loss of Chebychev 500kHz low pass filter with 1dB pass band ripple

To turn this low pass filter into a band pass filter, we resonate the shunt capacitors to the centre frequency with shunt inductors, and we resonate the series inductors to the centre frequency with series capacitors. The new circuit diagram and simulated response are shown in Figs. 5 and 6.

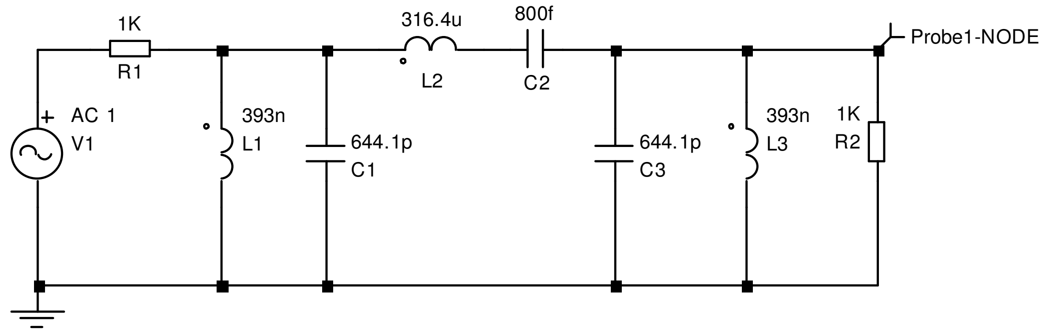

Fig 5. Band pass filter at 10MHz, 500kHz band width

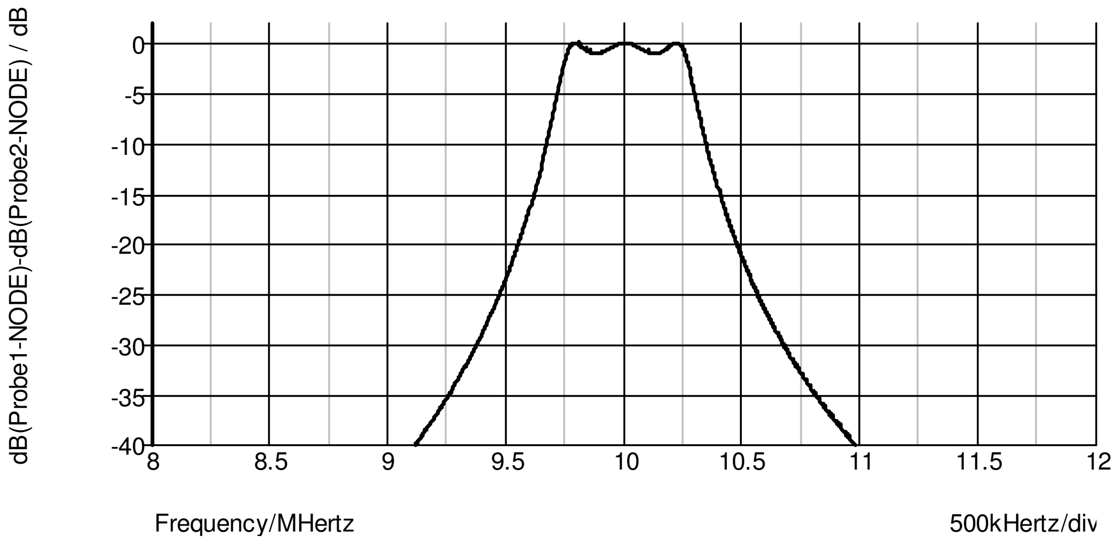

Fig 6. Simulated response of band pass filter shown in Fig 4

Here we meet the first problem. The series capacitor C2 is very small at 800fF (0.8pF), whilst the inductor is very large at 316.4uH. Is there some way that we can increase the series capacitance and reduce the series inductor value whilst retaining the same response? There is, and one solution is to undertake a circuit transformation by means of an "impedance inverter".

A well known example of an impedance inverter is the quarter-wave transmission line. If such a line, with a characteristic impedance of Zo, is terminated with impedance Z_{Load}, then the input impedance is given by:

$$Z_{IN} = Zo^2 / Z_{LOAD} \qquad (3)$$

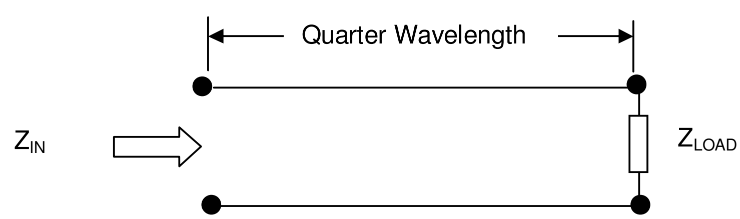

Fig 7. A quarter-wave transmission line impedance inverter

A quarter wave tranmission line makes a series tuned circuit at the design frequency connected to the far end look like a parallel tuned circuit when viewed from the input end.

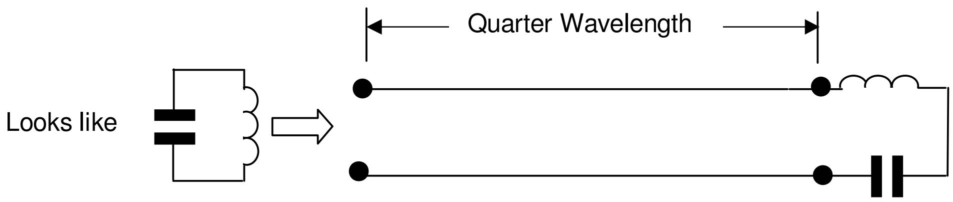

Fig.8. An application of an impedance inverter

There are discrete component equivalents of a quarter-wave transmission line, shown in Fig. 9.

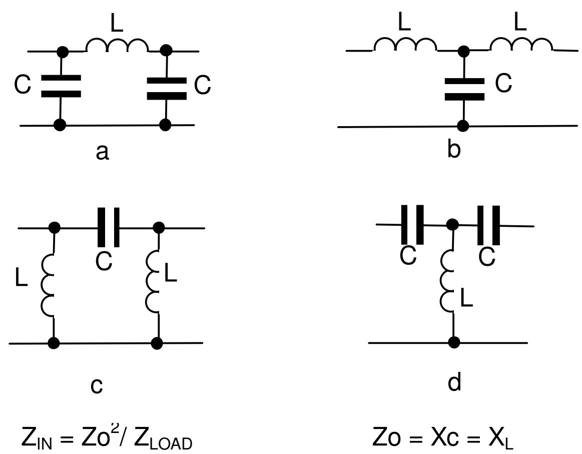

Fig. 9. LC Impedance inverters.

These only work perfectly at the frequency at which L and C are resonant. If the output of a perfect impedance inverter is shorted, then the input impedance is infinite. Taking Fig. 9a as an example, if the output capacitor is shorted, then L and the remaining C form a parallel tuned circuit, which, if the components are loss less, will have infinite impedance. Likewise, if no load is connected, i.e. an infinite impedance load, then L and the output capacitor form a series resonant circuit will have zero impedance if made with loss less components. The same short circuit/open circuit terminations can be applied to the other three configurations to show that they are also impedance inverters. Although we have considered the two extreme cases, circuit analysis shows that these circuits all act as impedance inverters irrespective of the load impedance but only at a single frequency. Is there some way that we can make an impedance inverter that works over a wide range of frequencies?

The impedance inverters shown in Fig. 9 work because the reactance of the inductor is equal and opposite to that of the capacitor at the resonant frequency. As the frequency is changed, the reactances change but in opposite directions, one becoming higher, the other lower. What we need is a component that has a reactance of the same sign as the inductor but with a reactance that moves in the same direction as a capacitor as the frequency changes. With such a component the impedance inverter will work over all frequencies. So what is this component? It is a negative capacitor. Does such a component exist? No, but that doesn’t stop us from using it if it is in parallel with a real capacitor large enough so that the sum of both capacitors is greater than zero. So if we want to use a -10pF capacitor in a filter and it is in parallel with a 250pF capacitor, then the combination can be replaced by a 240pF capacitor.

Wherever we have an inductor in Fig. 9, we can be replace it with a negative capacitor with the same numerical value as the “real” capacitor, for example

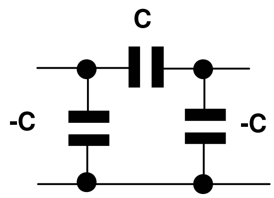

Fig. 10. An impedance inverter using negative capacitors.

If we terminate an impedance inverter with a parallel tuned circuit, then the input to the impedance inveter will look like a series tuned circuit. So using a pair of impendace inverters, we can replace the series tuned ciruit that couples the input and output parallel tuned circuits with impedance inverters and another parallel tuned cirrcuit.

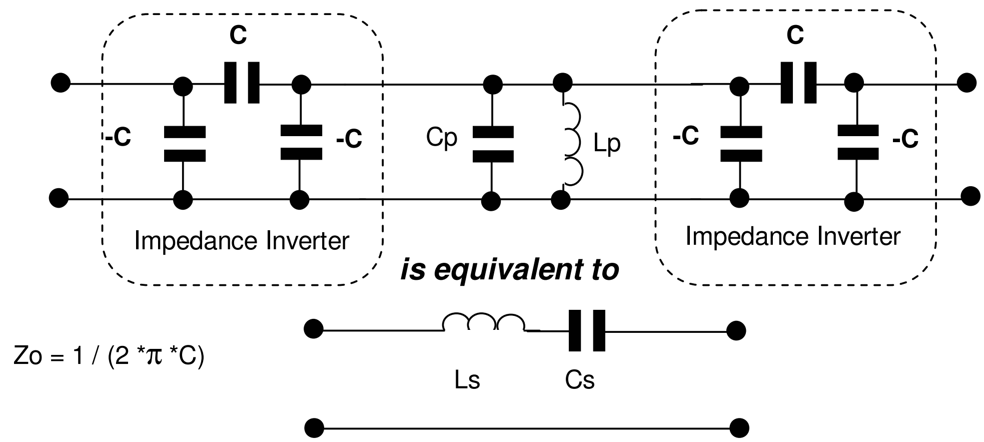

Fig 11. Replacing a series tuned circuit with a parallel tuned circuit

If Cp is greater than the sum of the two negative capacitors, then these can be absorbed into Cp. The negative capacitors at the ends can be absorbed into the remaining circuitry. The only remaining problem is to determine the value of C in the impedance inverters.

If we choose the value for C in the impedance inverter so that the inductor Lp of the parallel tuned circuit is the same as the inductors L1 and L3 at the input and the output ends of the filter, then only a single inductance value will be required for the filter. I won't go through the derivation of formula for calculating C – it's not difficult – but it turns out that it is the geometric mean of the shunt and series capacitors

$$C = \mathrm{SQRT}\ (C1\ C2) \qquad (4)$$

SQRT is EXCEL-speak for square root

From the values in Fig 5,

$$C = 22.7\mathrm{pF} \qquad (5)$$

For the impedance inverter

$$Zo = 1/(2\ \pi\ Fo\ C) \qquad (6)$$

The value of the capacitor Cp is given by

$$Cp = (2 \pi Fo C)^2 Ls \tag{7}$$

and from Fig 4

$$Cp = 644.1 \text{ pF}$$

and

$$Lp = 392 \text{ nH}$$

using the usual formula for tuned circuits because we set out to make its value the same as the end inductors. The circuit now becomes (drawn for convenience using the SPICE editor)

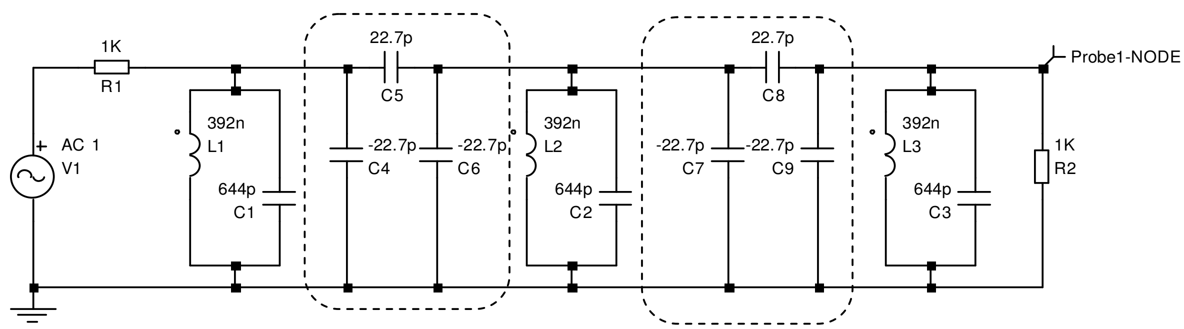

Fig. 12. Filter with impedance inverters

Now the negative capacitors, C4, C6, C7 and C9 can be absorbed into the larger (positive) capacitors giving the filter shown in Fig. 12.

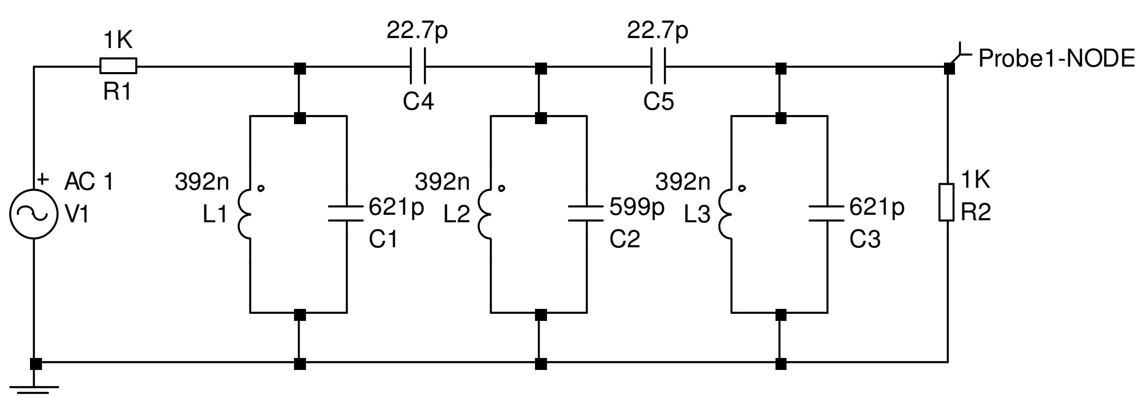

Fig. 13. Filter with negative capacitors absorbed.

This filter topology - tuned circuits with top coupling - may now look familiar as it is often encountered in RF band pass filter design and is often called “nodal” topology. It must be emphasised that the frequency response is identical to that of the filter in Fig 4 despite looking quite different.

A compact filter could be made using off-the-shelf axial inductors, such as may be obtained from RS Components, Farnell, CPC, Rapid Electronics etc, but the inductor value required for the above design is too small and, anyway, it is not a standard value. Also the losses, i.e. a poor Q, will have a detrimental effect on the performance. We will scale the filter to increase the value of the inductors to a standard value.

4. Impedance scaling filters

For the above filter, this is quite straightforward. If the terminating resistors are increased, then the inductors have to be increased proportionally, while the capacitors reduced proportionally. This ensures that the centre frequency remains unchanged.

If we want to use an off-the-shelf 1uH inductor for this filter, then the scaling factor is 1/0.393 = 2.54. So we multiply the terminating resistances by 2.54 and reduce all of the capacitors by the same factor. So we now have a filter with a standard inductor value.

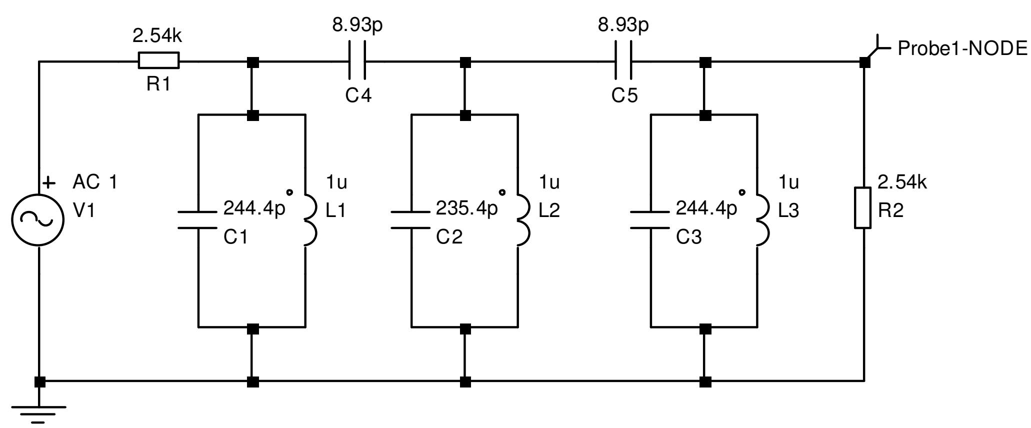

Fig. 14. Filter with scaled inductor value.

And the response is shown below in Fig. 15.

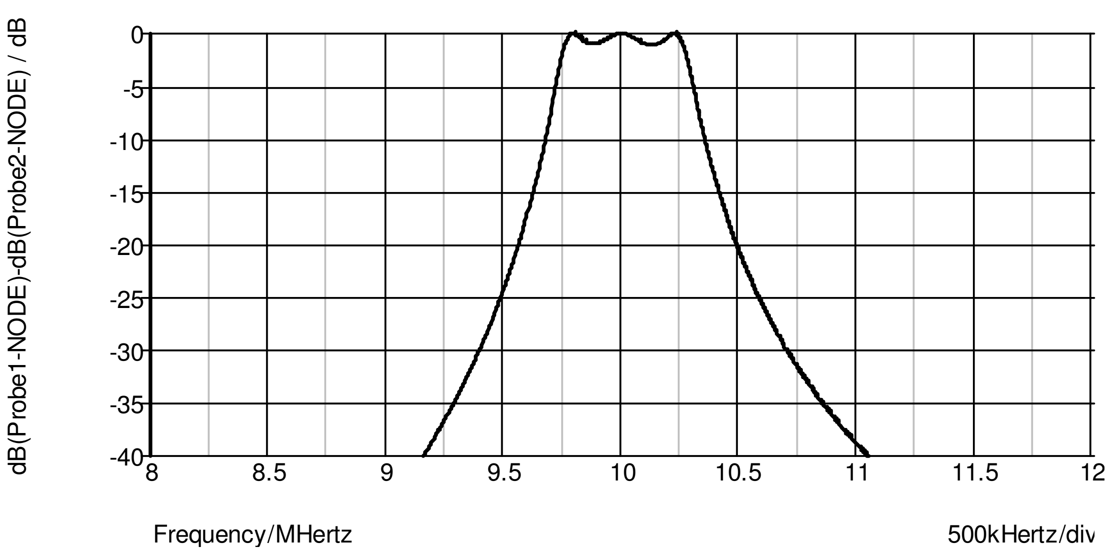

Fig 15. Filter insertion loss

The terminating resistors may be inconvenient for many applications and impedance matching will be discussed in a later part to this series.

5. Conclusion

The well known arrangement of top coupled parallel tuned circuits for band pass filtering can be derived from standard tables of normalised component values that are found in many text books. Central to the conversion of the standard filter configuration to top coupled tuned-circuits is the use of impedance inverters. Impedance scaling allows standard value inductors to be used. One or more parameters, such as terminating resistance, may have to be compromised to meet other more important criteria, such as the use of off-the-shelf inductors.

6. References and useful reference books

1. Jon B. Hagen, “Radio Frequency Electronics”, Cambridge. This is a very useful book dealing with most aspects of receiver and transmitter design. It has a two chapters dealing with filters.

2. Anatol I. Zverev, “Handbook of Filter Synthesis”, Wiley Interscience. This book was first published in 1967 and is regarded as a classic and is quoted as a reference in a great many articles about filter design. It is available new in paperback form.

3. Arthur B. Williams and Fred J, Taylor, ”Electronic Filter Design Handbook”, McGraw-Hill. This is a very good book, much more readable and usable than Zverev, at least for amateur radio projects.

4. Philip R. Geffe, “Simplified Modern Filter Design” Iliffe Books. First published in 1963, this is another classic. It has extensive tables of normalised component values for a wide range of filters. I have used a copy for 40 years (it cost 56 shillings, which dates it).

5. Yale J. Lubkin, “Filter Systems and Design: Electrical, Microwave and Digital”, Addison-Wesley Publishing. First published in 1970.

6. Erich Christian, “LC Filters Design, Testing and Manufacture”, Wiley Interscience. I borrowed a copy from the IET Library (also available to non-members) for the price of the return postage.

7. Hayward, Campbell and Larkin, “Experimental Methods in RF Design”. Published by the ARRL, this is a first-class book for the experimenter and may be purchased from the RSGB bookshop. Contains useful information about filter design including the alternative “q and k” method for designing band pass filters, which is described in Part 2 of this series. Highly recommended.

8. Wes Hayward W7ZOI, “Radio Frequency Design” published by the ARRL and available from the RSGB bookshop. Books by W7ZOI are useful, informative and understandable and this book is no exception. The book contains useful information on the alternative “q and k” method for designing band pass filters.

9. www.simetrix.co.uk. An excellent free evaluation version of SPICE.

10. www.aade.com. An excellent free filter design software for LC and crystal filters and used to support the sale of the AADE LC meter, which is capable of measuring small value inductors (less than 1uH) and capacitors (sub-pico-Farad resolution).

11. www.tonnesoftware.com. This website has a free student edition of ELSIE, an LC filter design program.

12. Gabor C. Temes & Sanjit K. Mita, “Modern Filter Theory and Design”, John Wiley. First published in 1973. Quite mathematical. However, like many such books you may find the odd piece of the filter jigsaw and this makes it worth dipping into when inspiration is required. The book contains extensive lists of references at the end of each chapter.

13. Cohn S. B. “Direct Coupled Resonator Filters”, Proceedings of the IRE, February 1957, pp 187 – 196.

Appendix 1
Tables of loss less low pass filter normalised component values

A1.1. Butterworth

The table below has been calculated from the formulae given by Cohn (Ref 13, Fig 2) but the results also appear in many publications. "N" is the order of the filter – that is the number of Ls and Cs. The bandwidth of the filter is the frequency at which the response has fallen by 3dB. The filters may start with a shunt capacitor or a series inductance as shown in Fig 1a or Fig. 1b respectively.

N	R_S	Shunt C G1	Series L G2	Shunt C G3	Series L G4	Shunt C G5	R_L
1	1.0000	2.0000					1.0000
2	1.0000	1.4142	1.4142				1.0000
3	1.0000	1.0000	2.0000	1.0000			1.0000
4	1.0000	0.7654	1.8478	1.8478	0.7654		1.0000
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000
N	R_S	G1 Series L	G2 Shunt C	G3 Series L	G4 Shunt C	G5 Series L	R_L

Table A1. Normalised component values for Butterworth low pass filters.

A1.2. Chebychev

Loss-less odd order Chebychev low pass filters are symmetrical and have equal input and output terminating resistances. The first element may be a series inductor or a shunt capacitor.

Even order Chebychev filters have different input and output terminating resistances. For even order Chebychev filters, the first element must be a series inductor. The different input and output terminating resistances mean gives the filter some voltage gain.

The tables below show the normalised component values for various orders of Chebychev response for pass band ripples of 0.1dB, 0.2dB, 0.5dB and 1dB. These are calculated from the formulae given in Ref. 13. An example of the difference between "Ripple bandwidth" and "3dB bandwidth" is shown in Fig. A1. This is a fixed ratio of each order of a filter.

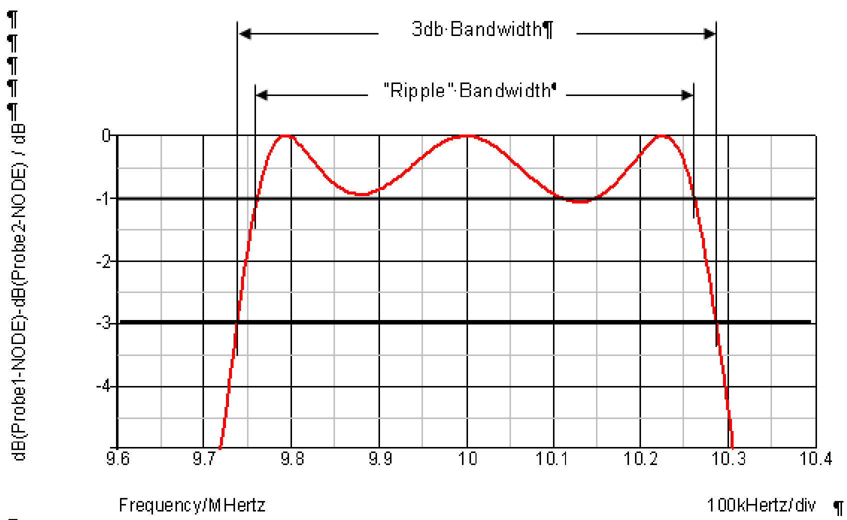

Fig. A1. Chebychev 1dB ripple filter showing "ripple bandwidth" and "3dB band width"

These tables give the component values for a bandwidth given by the frequency at which the response starts to falloff and is equal to the ripple amplitude.

Ripple dB	R_S	Shunt C G1	Series L G2	Shunt C G3	R_L	Voltage Gain (dB)
0.1	1.0000	1.0316	1.1474	1.0316	1.0000	0
0.2	1.0000	1.2276	1.1525	1.2276	1.0000	0
0.5	1.0000	1.5963	1.0967	1.5963	1.0000	0
1.0	1.0000	2.0237	0.9941	2.0237	1.0000	0
Ripple dB	**R_S**	**G1 Series L**	**G2 Shunt C**	**G3 Series L**	**R_L**	**Voltage Gain (dB)**

Table A2. Chebychev N=3 Normalised Component Values

Ripple dB	R_S	Series L G1	Shunt C G2	Series L G3	Shunt C G4	R_L	Voltage Gain (dB)
0.1	1.0000	1.1088	1.3062	1.7704	0.8181	1.3554	1.32
0.2	1.0000	1.3029	1.2844	1.9762	0.8468	1.5386	1.87
0.5	1.0000	1.6704	1.1925	2.3662	0.8419	1.9841	2.98
1.0	1.0000	2.0991	1.0644	2.8312	0.7892	2.6599	4.25

Table A3. Chebychev N=4 Normalised Component Values (NB G1 is a series inductor)

Ripple dB	R_S	Shunt C G1	Series L G2	Shunt C G3	Series L G4	Shunt C G5	R_L	Voltage Gain (dB)
0.1	1.0000	1.1468	1.3712	1.9750	1.3712	1.1468	1.0000	0
0.2	1.0000	1.3395	1.3370	2.1661	1.3370	1.3395	1.0000	0
0.5	1.0000	1.7058	1.2296	2.5409	1.2296	1.7058	1.0000	0
1.0	1.0000	2.1350	1.0911	3.0010	1.0911	2.1350	1.0000	0
Ripple dB	**R_S**	**G1 Series L**	**G2 Shunt C**	**G3 Series L**	**G4 Shunt C**	**G5 Series L**	**R_L**	**Voltage Gain (dB)**

Table A3. Chebychev N=5 Normalised Component Values

To convert the normalised component values to those which give a bandwidth given by the 3dB frequency, multiply the component values in the above tables by the factors given in Table A4.

N	$f3_{dB}/f_{ripple}$
3	1.0949
4	1.0530
5	1.0338

Table A4. Ratio of 3dB to ripple bandwidth

The following example shows how to do this. We will calculate the component values for a third order Chebychev filter with 1dB ripple and

Bandwidth Criteria	R_S	Shunt C G1	Series L G2	Shunt C G3	R_L	Notes
Ripple (-1dB)	1.0000	2.0237	0.9941	2.0237	1.0000	From Table A2
-3dB response	1.0000	2.2157	1.0884	2.2157	1.0000	Multiplied by factor from Table A4 for N=3

Table A5. Normalised component values for 1dB (ripple) bandwidth and 3dB bandwidth

The normalised frequency is 1 radian/sec, which corresponds to 1/(2π) Hz, or 0.1592Hz. Fig 1A shows the responses of both versions of the filter in the vicinity of 0.1592Hz, as simulated by SPICE (Ref. 9).

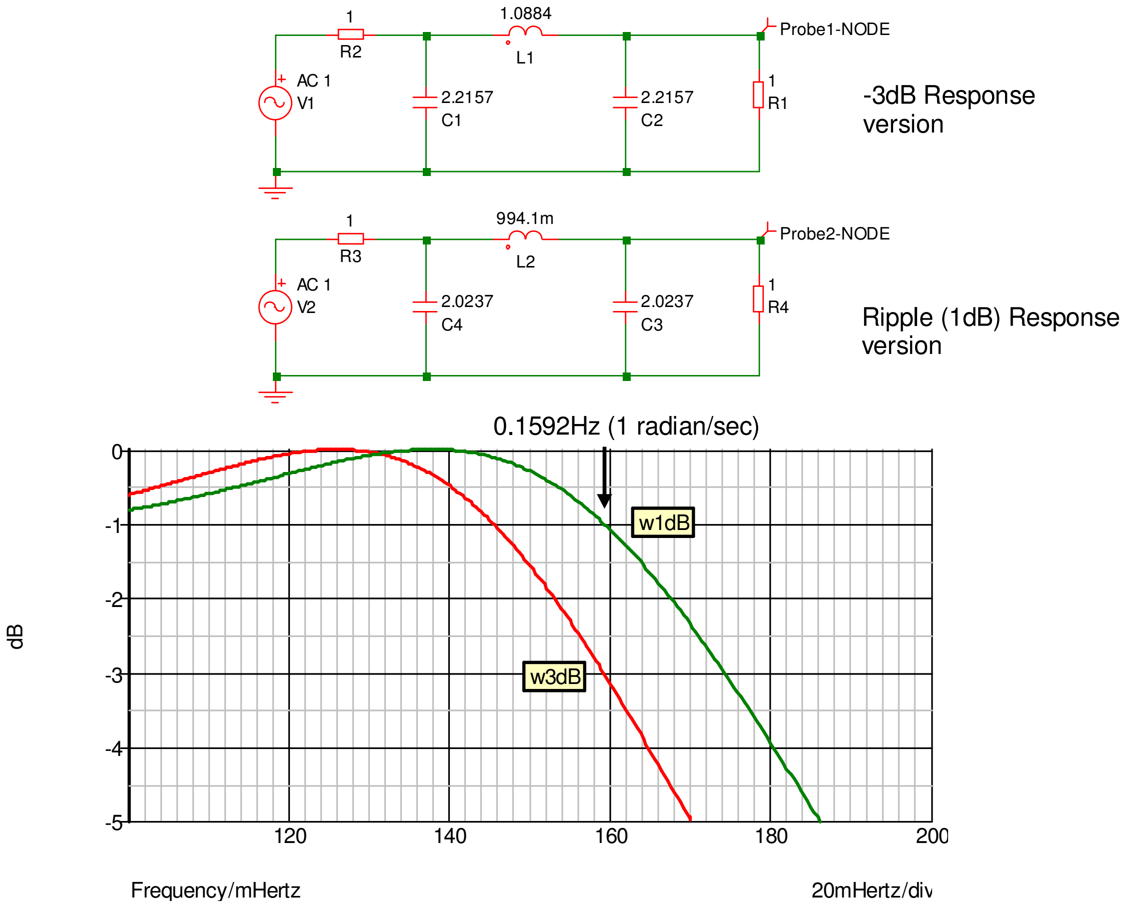

Fig. A2.   Response of filters with cut off frequencies equal to the ripple and 3dB responses

So which of these two responses should you use? It all depends…. The basic Chebychev response defines the bandwidth to the point at which the response has fallen off to a level equal to the ripple. On the other hand, if comparing Chebychev filters with other types, for example Butterworth, then it may be better to use the 3dB bandwidth version.

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July 2010
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