Band Pass Filter Design
Part 2. Band pass filter design using the “q & k” method

1. Introduction

Part 1 of this series showed how a familiar band pass filter topology can be derived from a basic low pass filter prototype. There is an alternative method of designing narrow band pass filters, again using component values from pre-calculated tables and scaling to the centre frequency, band width and terminating resistance

2. The “q & k” method of band pass filter design – two coupled resonators

Fig. 1 shows the familiar filter topology and which appears as Fig. 14 in part 1 of this series.

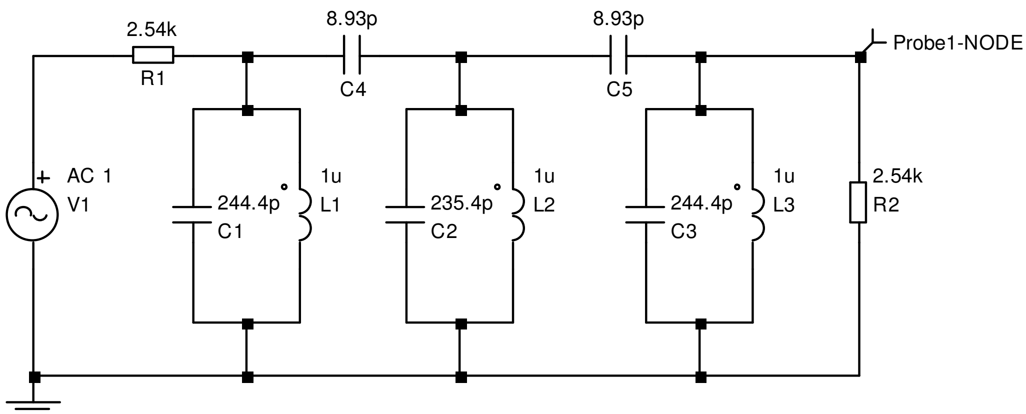

Fig 1. 10MHz filter, 500kHz bandwidth, Chebychev response with1dB ripple

This topology, often referred to as “nodal” topology, consists of a series of tuned circuits, or “resonators”, top-coupled by means of small value capacitors. The justification for the alternative “q & k” method is that the coupling elements have constant impedance with frequency – this will be approximately true over a narrow bandwidth, say less that 10% of the centre frequency. Although inductors could be used as coupling elements, in practice these coupling elements are invariably capacitors. Coupling capacitors introduce additional attenuation on the low frequency side of the filter response and make the filter response unsymmetrical. As will be shown in another part to this series, the design can be further altered so that the response within the pass band is unchanged but the response on the high frequency side is much improved at the expense of the low frequency side. In many instances, this is a very acceptable trade-off because of the need to attenuate harmonics of a signal.

Most of the circuit diagrams that appear in this series are copied from the SIMetrix SPICE simulator program - this is an excellent Windows based simulator and a fully functioning free evaluation version is available free of charge from the web site (Ref. 1). However, there are some limitations to the naming of components and sometimes the circuit diagram is better drawn using WORD.

The “q and k” values are obtained from tables, rather like the normalised values for low pass filters, but are much more convenient for band pass filters. The “q” values are used to determine the loaded Qs of the end resonators and the “k” values are used to calculate the coupling capacitors between resonators.

Fig. 2 shows a band pass filter based on two tuned circuits that are "top-coupled" by a capacitor.

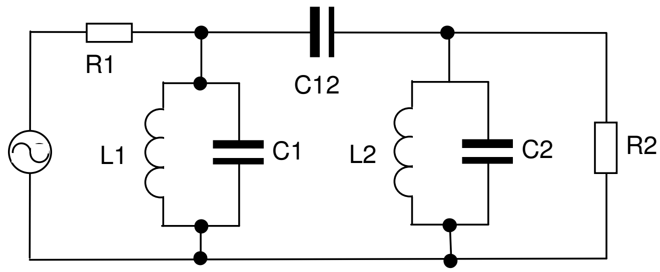

Fig. 2. Band pass filter with two tuned circuits.

Tables of "q" and "k" are available from many books (Refs 2, 3, 4 and 5) and these are usually calculated for a 3dB bandwidth, including those for Chebychev filters. In Part 1 we used the standard definition of bandwidth for Chebychev filters, the frequency at which the response has fallen to a value equal to the ripple, so if the filter had been designed to have 1dB of ripple, then the bandwidth would be the 1dB bandwidth. There is a formula to convert between the 3dB bandwidth and the "ripple" bandwidth of Chebychev filters.

The q and k values for a two-resonator filter are shown below. Other values are available for up to eight resonators, although component tolerances and inductor losses would probably make filters with more than three resonators difficult to construct.

Filter Type	q1	k12	q2
Butterworth	1.414	0.707	1.414
Chebychev 0.1dB ripple	1.638	0.711	1.638
Chebychev 0.5dB ripple	1.950	0.723	1.950
Chebychev 1dB ripple	2.210	0.739	2.210
Bessel	0.5755	0.900	2.148

Table 1. q and k values for two coupled resonator

The table of values is used as follows:

1. Choose the filter type – for this exercise we will choose the Chebychev 0.1dB response – and the centre frequency and bandwidth. **NB** the bandwidth should be no more than 10% of the centre frequency. For this exercise we will design a filter with a bandwidth of 500kHz centred on 10MHz and with inductor values of 1uH – the calculated values are shown in brackets

2. Calculate the pass band "Q", Q_{BP}

$$Q_{BP} = Fo / BW_{3dB} \qquad (Qbp = 20)$$

Where Fo is the filter centre frequency and BW_{3dB} is the desired 3dB bandwidth.

2. Calculate the loaded Q for input and output resonator, Q1 and Q2 as follows:

$$Q1 = Q_{BP} * q1 \qquad (Q1 = 32.76)$$

$$Q2 = Q_{BP} * q2 \qquad (Q2 = 32.76)$$

For many filters Q1 and Q2 will be the same value

3. Calculate the coupling coefficient K12 for this filter as follows:

$$K12 = k12 / Q_{BP} \qquad (K12 = 0.0356)$$

4. Choose a convenient value for the inductor. This may be a standard value or perhaps the value for a home wound inductor. The input and output terminating resistors R1 and R2 can now be calculated

$$R1 = 2 \pi Fo\ L\ Q1 \qquad (R1 = 2.058k)$$

$$R2 = 2 \pi Fo\ L\ Q2 \qquad (R1 = 2.058k)$$

For many filters R1 and R2 will be the same value

5. Calculate the nodal capacitance, Cnode, which resonates with the inductor at Fo

$$Cnode = \frac{1}{(2 \pi Fo)^2 L} \qquad (Cnode = 253.3pF)$$

6. Calculate the coupling capacitor C12

$$C12 = K12 * Cnode \qquad (C12 = 9.02pF)$$

7. Finally the values of C1 and C2 can be calculated. These are the capacitors that resonate with the inductors at Fo, less the value of the coupling capacitor C12

$$C1 = Cnode – C12 \qquad (C1 = 244.3pF)$$

$$C2 = Cnode – C12 \qquad (C2 = 244.3pF)$$

The final circuit diagram is shown in Fig. 3.

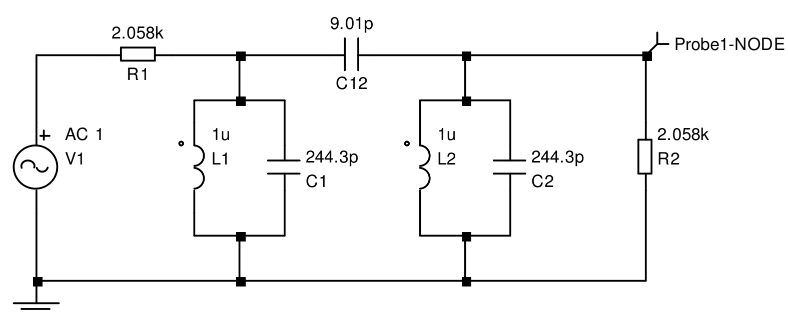

Fig 3. Band pass filter, centre frequency 10MHz, 500kHz bandwidth 0.1dB ripple

So what does the filter and the response look like? We can use SPICE to calculate the insertion loss

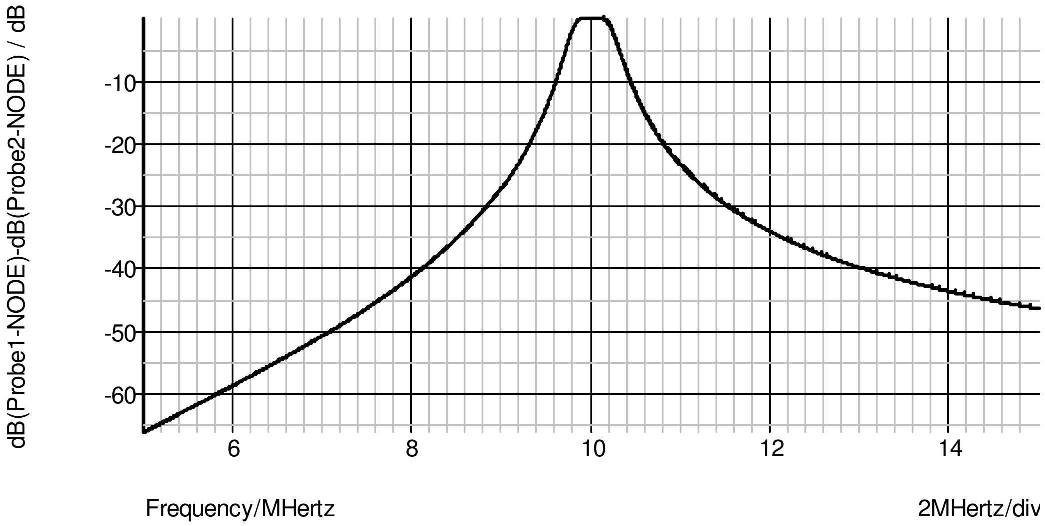

Fig. 4. Insertion loss of filter

The attenuation on HF side falls off more slowly than on the LF side. The HF response can be improved as will be shown later in this series.

An expanded view of the pass band is shown in Fig. 5.

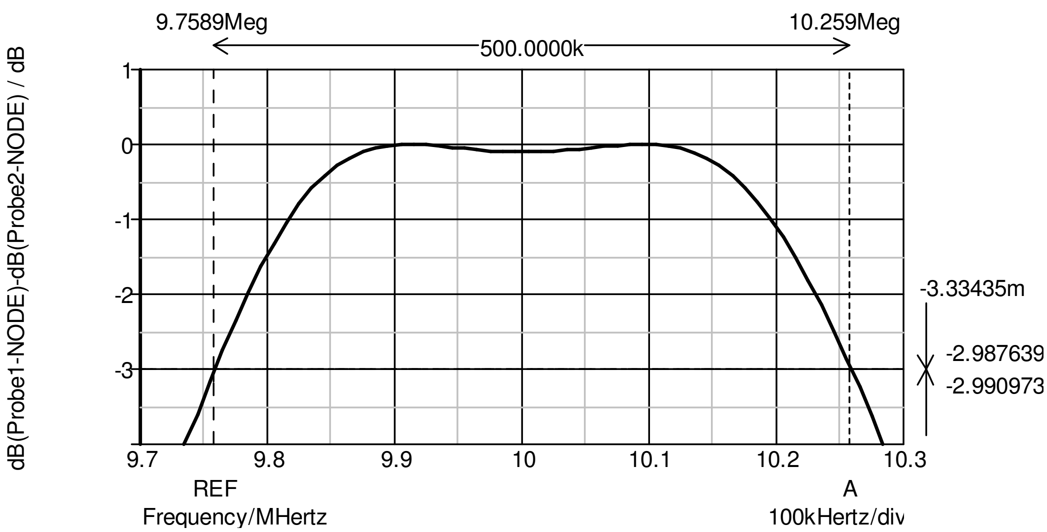

Fig. 5. Insertion loss expanded to show pass band

There is a very slight amount of pass band ripple – the specification is for 0.1dB. The 3dB bandwidth, shown by the cursors, is exactly 500kHz

A real filter will not have exactly this response because of component tolerances. So in the above design, we could make C1 and C2 = 242p (220 + 12 + 10pF), C12 = 10pF and R1 and R2 = 2k. SPICE will now give us a comparison with the ideal values.

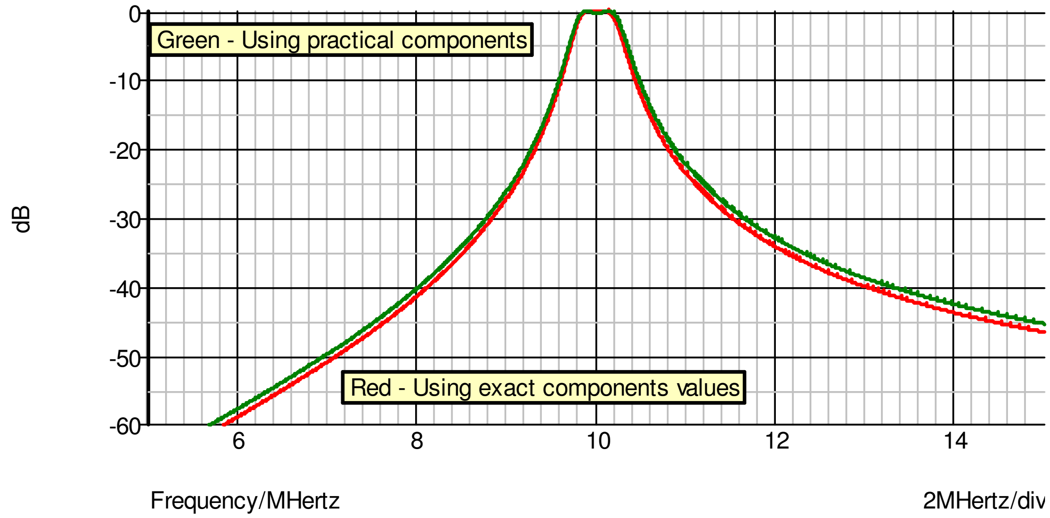

Fig 6. Insertion loss of comparison of filter with practical component values

The pass band ripple is determined mainly by the values of the terminating resistors. Fig. 7 shows how the insertion loss of the filter changes with different values of terminating resistor.

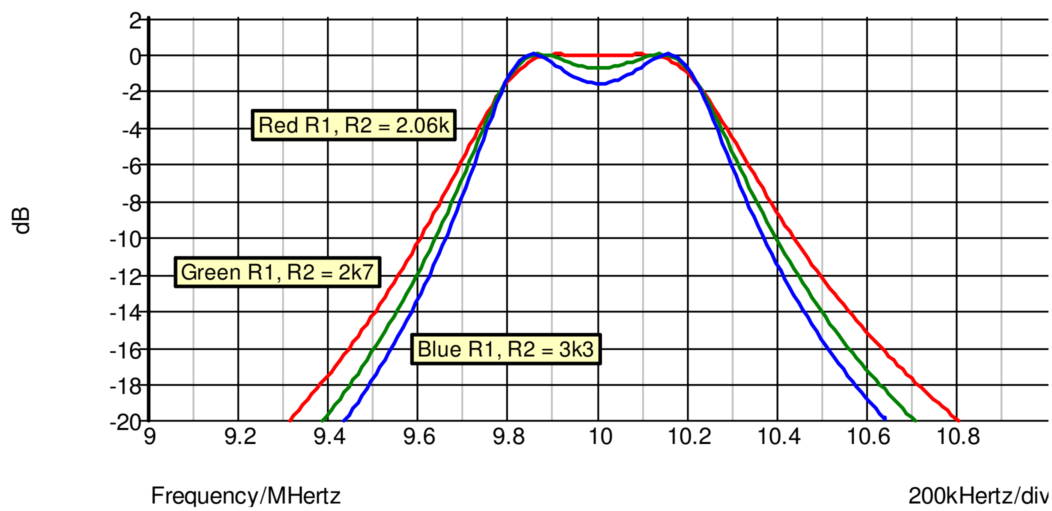

Fig 7. The effect of changing the terminating resistors on insertion loss

The centre bandwidth is little changed but as R1 and R2 are increased in value the ripple becomes more pronounced. On the other hand, the response fall off more quickly and so there is a compromise between pass band ripple and rate of roll off of the insertion loss response.

However, we have not yet taken into account the losses in components, particularly the inductors. The effects of inductor losses will be examined in a later part to this series.

3. Three coupled resonators

The “q & k” method can be extended to three resonators.

Filter Type	q1	k12	k23	q3
Butterworth	1	0.707	0.707	1
Chebychev 0.1dB ripple	1.433	0.662	0.662	11.433
Chebychev 0.5dB ripple	1.864	0.647	0.647	1.864
Chebychev 1dB ripple	2.210	0.638	0.638	2.210
Bessel	0.337	1.748	0.684	2.203

Table 2. q and k values for three coupled resonator

1. Choose the filter type – for this exercise we will choose the Chebychev response with 0.5dB – and the centre frequency and bandwidth. **NB** the bandwidth should be no more than 10% of the centre frequency. For this exercise we will design a filter with a bandwidth of 350kHz centred on 3.65MHz and with inductor values of 4.7uH – the calculated values are shown in brackets

2. Calculate the pass band Q_{BP}

$$Q_{BP} = Fo / BW_{3dB} \qquad (Q_{BP} = 10.43)$$

Where Fo is the filter centre frequency and BW_{3dB} is the desired 3dB bandwidth.

2. Calculate the loaded Q for input and output resonator, Q1 and Q3 as follows:

$$Q1 = Q_{BP} * q1 \qquad (Q1 = 19.44)$$

$$Q3 = Q_{BP} * q3 \qquad (Q3 = 19.44)$$

For many filters Q1 and Q3 will be the same value

3. Calculate the coupling coefficient K12 and K23 for this filter as follows:

$$K12 = k12 / Q_{BP} \qquad (K12 = 0.0620)$$

$$K23 = k23 / Q_{BP} \qquad (K23 = 0.0620)$$

4. Choose a convenient value for the inductor. This may be a standard value or perhaps the value for a home wound inductor. The input and output terminating resistors R1 and R2 can now be calculated

$$R1 = 2 \pi \text{ Fo L Q1} \qquad (R1 = 2{,}095 \text{ Ohms})$$

$$R2 = 2 \pi \text{ Fo L Q3} \qquad (R1 = 2{,}095 \text{ Ohms})$$

For many filters R1 and R2 will be the same value

5. Calculate the nodal capacitance, Cnode, which resonates with the inductor at Fo

$$Cnode = \frac{1}{(2 \pi Fo)^2 L} \qquad (Cnode = 404.5pF)$$

6. Calculate the coupling capacitor C12

C12 = K12 * Cnode (C12 = 25.1pF)

C23 = K23 * Cnode (C23 = 25.1pF)

7. Finally the values of C1 and C2 can be calculated. These are the capacitors that resonate with the inductors at Fo, less the value of the coupling capacitor C12

C1 = Cnode – C12 (C1 = 379.2pF)

C2 = Cnode – C12 – C23 (C2 = 354.1pF)

C3 = Cnode – C12 (C2 = 379.2pF)

Note that C2 is detuned by both C12 and C23, so C2 has to be adjusted appropriately. The final circuit is shown in Fig. 8.

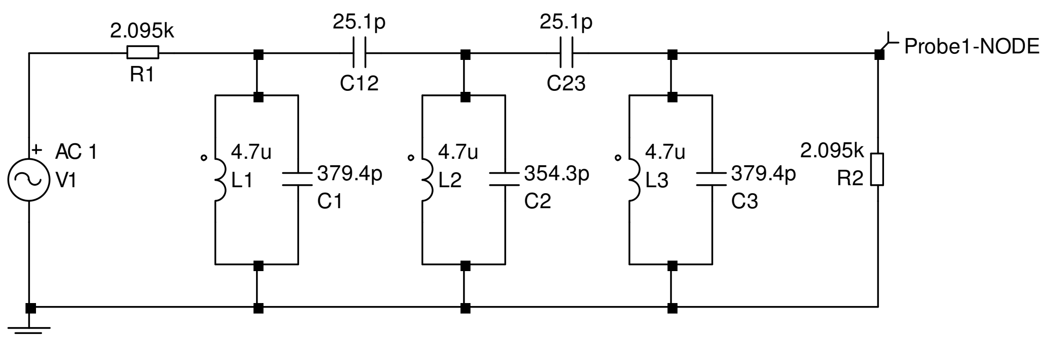

Fig. 8. 3.65 MHz, 350kHz bandwidth filter circuit

The insertion loss is shown in Fig. 9. Again, the roll-off on the LF side is better than on the HF side. An expanded view of the pass band response is shown in Fig. 10.

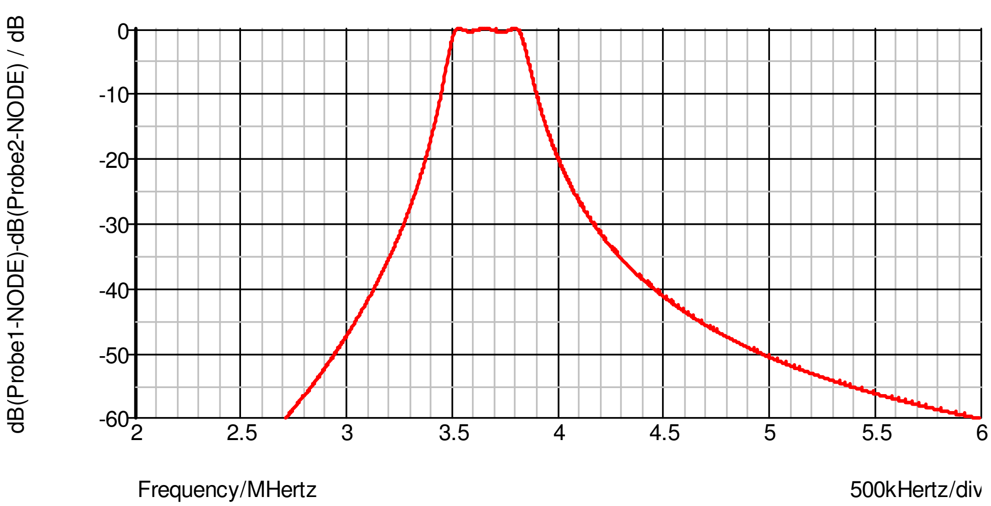

Fig. 9. Insertion loss of the filter in Fig. 8.

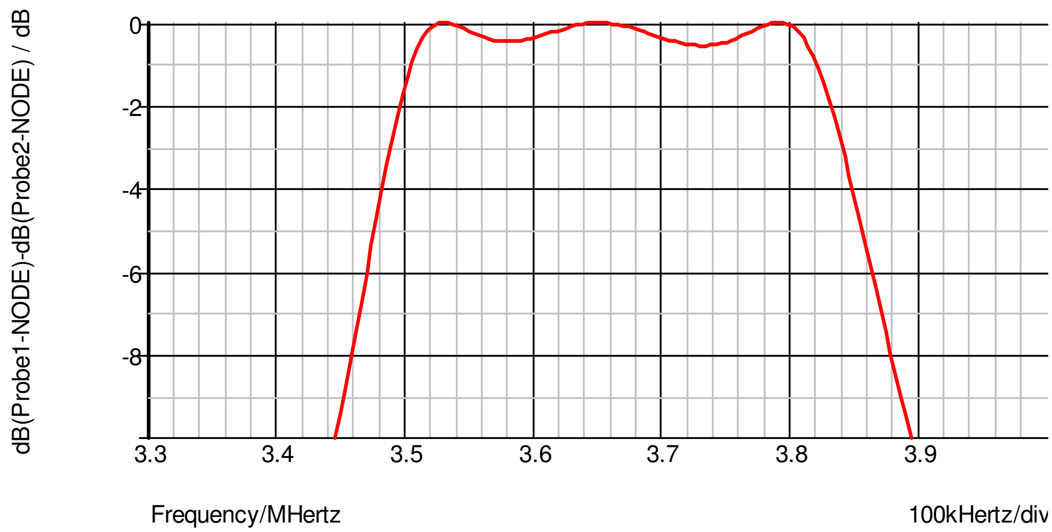

Fig. 10. Pass band insertion loss.

The standard tables of normalised values for low pass filters that are the basis for Part 1 of this series are derived from the mathematical relationship between the input and the output of the filter. So how are the "q" and "k" values in the tables given in this Part derived? I have been able to calculate these from the normalised low pass filter values but only for an odd number of resonators. Tables exist for filters with odd and even numbers of resonators, so I am still investigating how I can derive the values for an even number of resonators.

Although most capacitors used in filters are low loss, the same cannot be said of inductors. If an estimate of the Q of each inductor can be made at the filter centre frequency, then SPICE can be used to predict the frequency response of a practical filter. The subject of inductor losses will be covered in Part 6 of this series of articles.

4. Conclusion

An alternative method of band pass filter design has been discussed, which is much easier to use than developing a band pass filter from the standard tables for low pass filters. Tables of "q & k" values are available from a variety of books. A simple spreadsheet can be composed to calculate the component values and the theoretical response verified by use of a SPICE simulator.

5. References

1. www.simetrix.co.uk An excellent free evaluation version of the SPICE simulator.

2. Anatol I. Zverev, "Handbook of Filter Synthesis", Wiley Interscience. This book was first published in 1967 and is regarded as a classic and is quoted as a reference in a great many articles on filter design. It is now available new in paperback form.

3. Arthur B. Williams and Fred J, Taylor, "Electronic Filter Design Handbook", McGraw-Hill. This is a very good book, much more readable and usable than Zverev, at least for amateur radio projects.

4. Hayward, Campbell and Larkin, “Experimental Methods in RF Design”. Published by the ARRL, this is a first-class book for the experimenter and may be purchased from the RSGB bookshop. Highly recommended.

5. Wes Hayward W7ZOI, “Radio Frequency Design” published by the ARRL and available from the RSGB bookshop. Books by W7ZOI are useful, informative and understandable and this book is no exception.

Appendix A1.
Values of q & k for Butterworth and Chebychev filters

Values of q &k for Butterworth and Chebychev filters are shown below. “N” is the number of resonators. These are taken from Ref. 2.

N	q1	q2	k12	k23	k34	k45
2	1.4142	1.4142	0.7071			
3	1.0000	1.0000	0.7071	0.7071		
4	0.7654	0.7654	0.8409	0.5412	0.8409	
5	0.6180	0.6180	1.0000	0.5559	0.5559	1.0000

Table A1. q & k values for Butterworth Filters with two to five resonators

Ripple dB	q1	q2	k12
0.01	1.4829	1.4829	0.7075
0.1	1.6382	1.6382	0.7106
0.5	1.9497	1.9497	0.7225

Table A2. q & k values for two resonator Chebychev filters

Ripple dB	q1	q2	k12	k23
0.01	1.1811	1.1811	0.6818	0.6818
0.1	1.4328	1.4328	0.6618	0.6618
0.5	1.8636	1.8636	0.6474	0.6474

Table A2. q & k values for three resonator Chebychev filters

Ripple dB	q1	q2	k12	k23	k34
0.01	1.0457	1.0457	0.7369	0.5413	0.7369
0.1	1.3451	1.3451	0.6850	0.5421	0.6850
0.5	1.8258	1.8258	0.6482	0.5446	0.6482

Table A1. q & k values for four resonator Chebychev filters

Ripple dB	q1	q2	k12	k23	k34	k45
0.01	0.9766	0.9766	0.7796	0.5398	0.5398	0.7796
0.1	1.3013	1.3013	0.7028	0.5355	0.5355	0.7028
0.5	1.8086	1.8068	0.6519	0.5341	0.5341	0.6519

Table A1. q & k values for five resonator Chebychev filters

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