Band pass filter design
Part 5 - Bartlett's Bi-section Theorem

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1. Introduction.

In the Part 3 Norton's Transform was used to change the impedance level within a filter so that the terminating resistance could be changed. There is an alternative method called Bartlett's Bisection Theorem, which can be used for filters that have symmetrical circuit diagrams.

2. Using Bartlett's Bi-section Theorem

We will look at a filter with the following specification:

Centre Frequency 10MHz
Bandwidth 500kHz
Response Chebychev with 1dB ripple
Two tuned circuits, top coupled
Loss-less components

The losses due to "real" components, mainly the finite "Q" of the inductors, can be assessed using the SPICE simulator (Ref. 4). For the purposes of this exercise we will assume that the components are loss-less.

The filter can be designed from first principles as outlined in Part 1, or the "q k" method outlined in Part 2 (Refs 1,2 and 3). Fig. 1 shows the filter (drawn using the SPICE simulator) – note that the terminating resistance cannot be chosen independently – it's one of the outputs of the design and a matching section may be required to transform this resistance to the required terminating resistance.

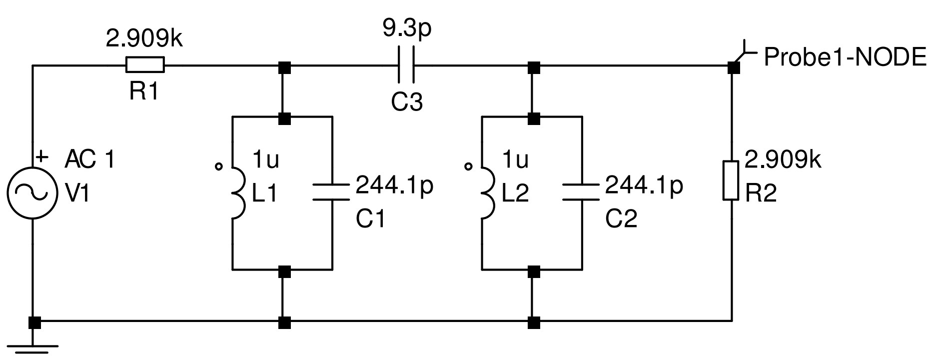

Fig 1. Filter circuit

This filter is symmetrical about C3 but it must be split into two parts so that the filter is symmetrical on either side of a line through the middle of the filter. For the purposes of determining whether a filter is symmetrical the voltage source on the left can be replaced with a short circuit. A modified circuit is shown in Fig. 3 where C3 now consists of C3a and C3b, each twice the value of the original C3.

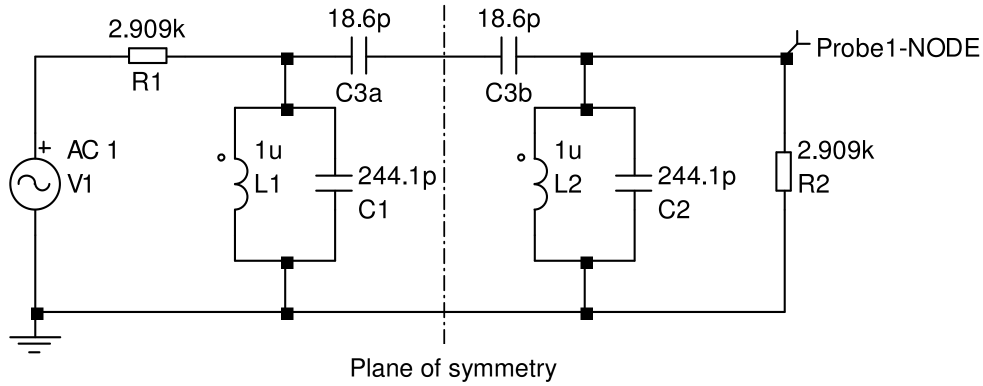

Fig 2. Filter modified for symmetry about the middle of the filter

Now we have a filter that is symmetrical. The impedance level part way through the filter can now be changed by multiplying all of the resistors and inductors on one side of the plane of symmetry by a constant and dividing the capacitors by the same constant.

If we stay with standard value inductors, we will change the impedance level on the right hand side of the filter by a factor of 2.2. This is shown in Fig. 3.

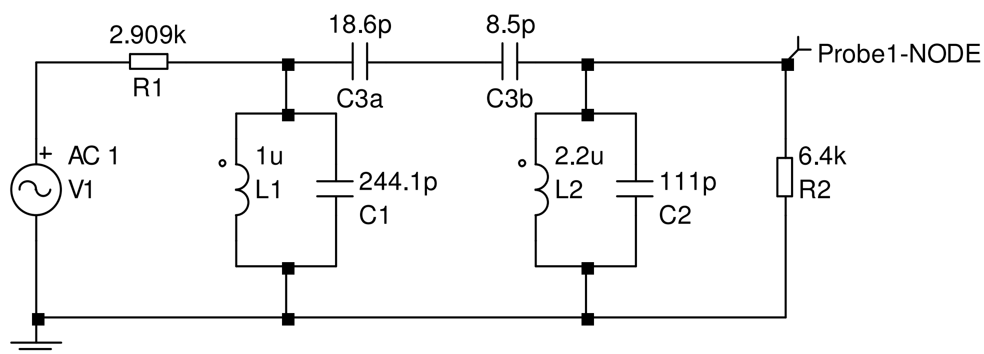

Fig 3. Filter with impedance level on right hand side increased.

Now C3a and C3b can be re-combined using the usual capacitors-in-series formula

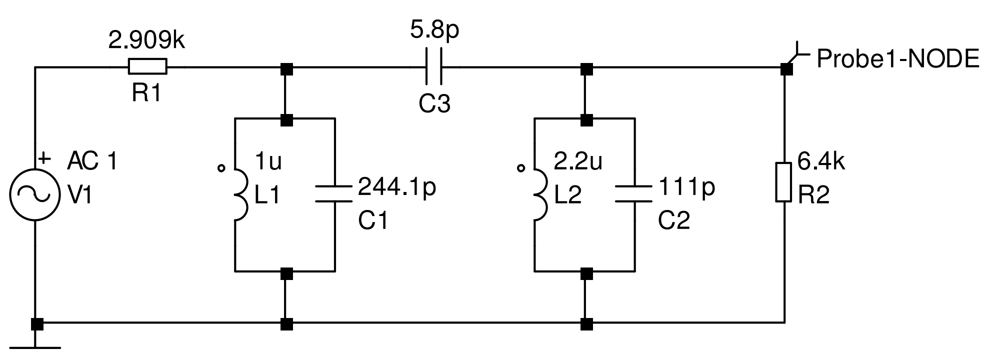

Fig 4. Final filter design

There is a difference between a filter design with an impedance change using Bartlett's Bisection Theorem and design using Norton's transform and this can be seen using the SPICE simulator. Fig 5 shows the pass band of filters with the same input and output terminating resistances designed using Norton's Transform and Bartlett's Bi-section Theorem. Both show a modest insertion gain due to the higher output terminating resistance but the version using Bartlett's Theorem has a lower insertion gain by about 0.6dB. Norton's transform gives a filter with maximum power transfer. This is not so when Bartlett's Theorem is used and so there is an additional loss due to this mismatch.

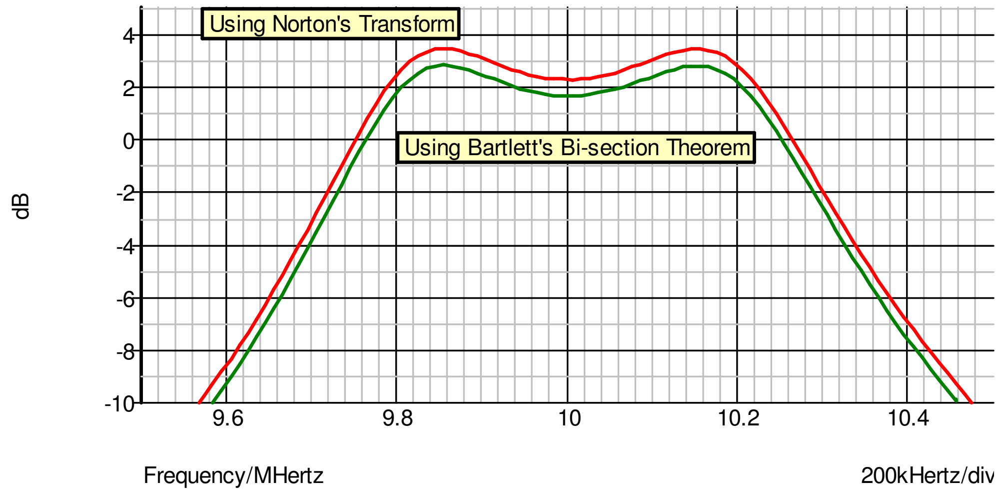

Fig. 5. Comparison of insertion loss using Norton's Transform and Bartlett's Theorem

3. Splitting the middle component to give symmetry

Although a filter may have a symmetrical circuit diagram, it will probably require some work to get it into a form suitable to apply Bartlett's Theorem, as we saw above. Some of the situations that you may meet for the middle components of the filter, and the solutions are shown below

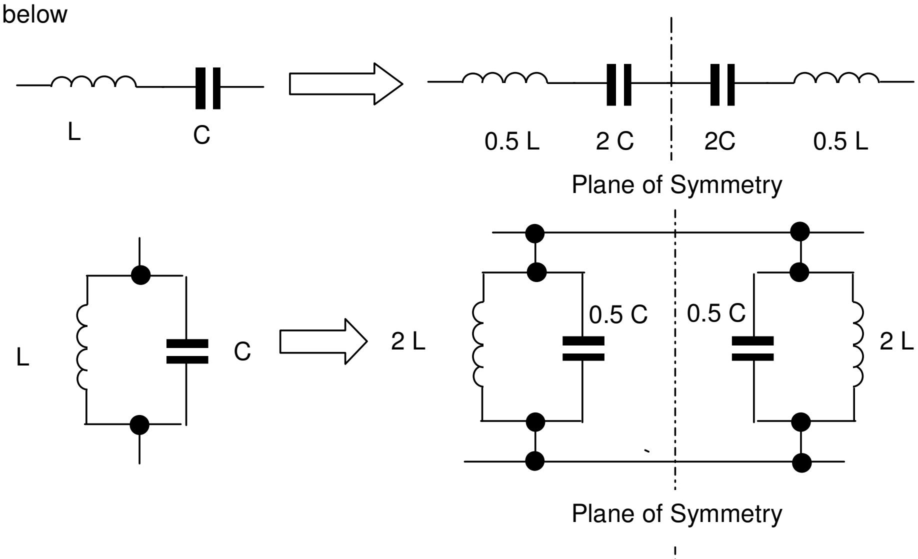

Fig. 6. Converting the middle component so that Bartlett's Theorem can be used

Once the circuit diagram is in a suitable symmetrical form, then Bartlett's Theorem can be applied and the components on one half of the filter multiplied/divided by a constant depending on whether they are inductors, resistors or capacitors.

4. Conclusion

Bartlett's Bi-section Theorem is a useful method of increasing or decreasing filter impedance so as to match a different terminating resistance. It is only applicable to filters that are symmetrical about the middle component(s) of the circuit diagram. The shape of the filter response is unchanged after undergoing transformation.

5. References

1. Wes Hayward W7ZOI, "Radio Frequency Design" published by the ARRL and available from the RSGB bookshop.

2. Hayward, Campbell and Larkin, "Experimental Methods in RF Design". Published by the ARRL, this is a first-class book for the experimenter and may be purchased from the RSGB bookshop.

3. William E Sabin W0IYH, "Narrow Band-pass Filters for HF", QEX Sep/Oct 2000 pp13 – 17.

4. www.simetrix.co.uk An excellent free fully functioning evaluation version of SPICE.

Other useful references are given in Part 1.

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