Band pass filter design
Part 6. Losses in inductors

1. Introduction

In Part 6 of this series, we will look at the effects of losses in inductors upon the insertion loss of a filter. A Chebychev 10MHz two-resonator filter with a 500kHz bandwidth and 1dB pass band ripple will be examined. Two inductors will be considered, an off-the-shelf 1uH inductor with a Q of 70 at 10MHz (e.g. Farnell Electronics part number 164-4248) and a toroid-wound inductor of the same inductance and with a Q of 200. The effect of the losses of these two inductor will be compared to that of lossless inductors. The insertion loss and degradation of the response caused by the losses in the inductors will be explored using a SPICE simulator (Ref. 1).

2. Losses in inductors

Losses in components make themselves felt by degrading the pass band response and so we will only be looking at a 1MHz wide band centred on the nominal frequency of the filter. The shape of the stop band is usually not significantly affected.

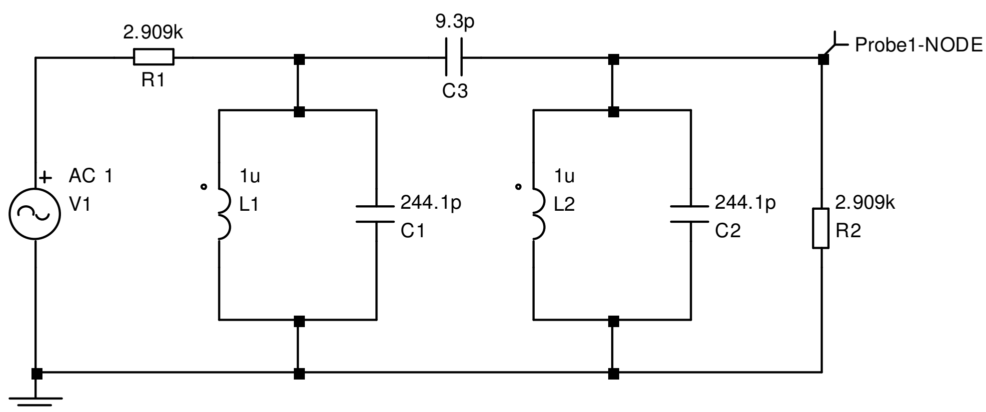

Fig. 1. 10MHz band pass filter, 500kHz bandwidth, Chebychev response, 1dB ripple

The losses in the inductors are simulated by adding resistors in series, with values given by

$$Rs = \frac{2\,\pi\ Fo\ L}{Q_{F0}}$$

Where Q_{F0} is the inductor Q at frequency Fo. The d.c. resistance will be lower than Rs – the resistance increases with frequency due to the skin effect. For a 1uH inductor with a Q of 70 at 10MHz, the series resistance is 0.90 Ohms, and for an inductor with a Q of 200 the series resistance will be 0.314 Ohms. Fig. 2 shows the effect of the loss in the inductors. Notice that the response becomes more rounded.

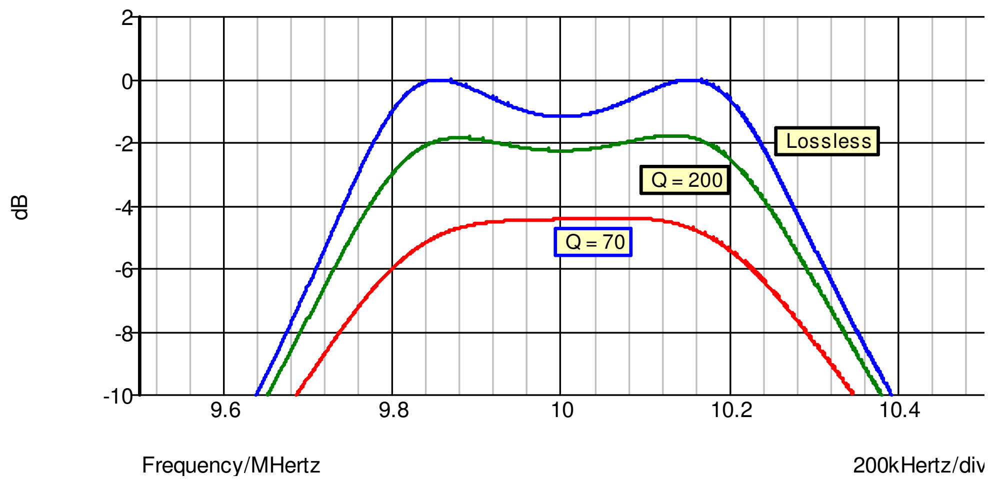

Fig 2. Insertion loss of the filter in Fig 1 with lossless inductors

The edges of the pass band are badly affected. Why is this so? Again SPICE comes to our aid because the currents in components can also be displayed. Fig. 3 shows the currents in L1 and L2 for lossless inductors and a 1V signal generator.

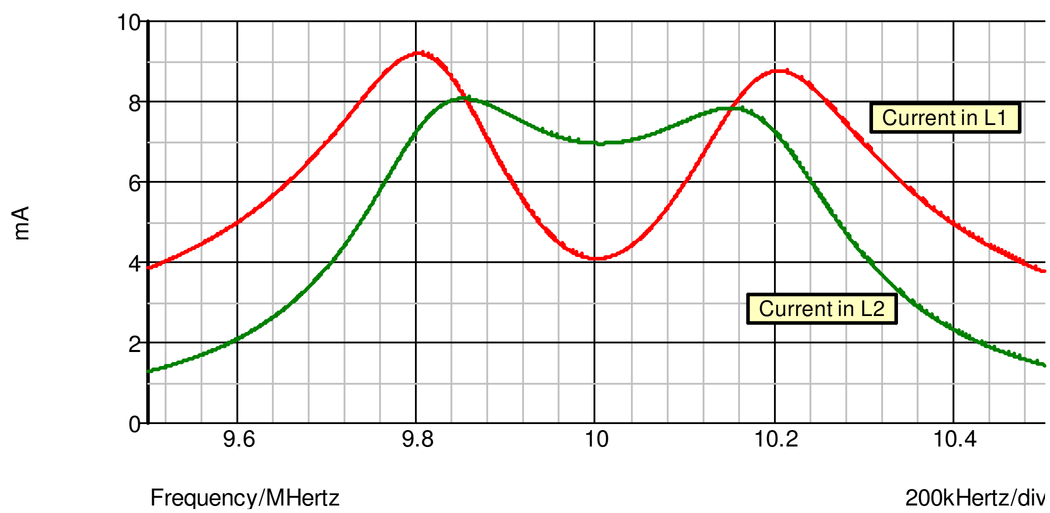

Fig. 3. Current in L1 and L2

The currents in the inductors peak towards the edges of the pass band. Recall that the power dissipated in a resistor is proportional to the square of the current, so at frequencies of 9.8 MHz and 10.2 MHz over five times as much power is dissipated compared with that at a frequency of 10.0 MHz. Therefore the losses towards the edges of the pass band increase significantly and give the rounding of the pass band. The current is falling off quite rapidly outside of the pass band and so the effects of losses will be less in the stop band.

The variation in current in L2 is much less and this is true of band pass filters as you progress from input to output.

Must we accept less than theoretical performance when we construct practical filters? There are a couple of ways in which we can improve the performance of practical band pass filters. One is to deliberately add loss to resonators and used revised standard tables of components for the design. This will be examined in Part 8 of this series and is capable of giving the calculated response, albeit with some additional loss. Another technique that can be used is to increase the input and output terminating resistors so that the combination of the revised terminating resistors and the resistive losses in the inductors is the same the terminating resistors of the loss-less design. We will now look at this latter technique.

3. Compensating for inductor losses

If the inductor has a reasonable Q, say >10, then the losses in the inductor can be approximated by either series or parallel resistance over a limited range of frequencies around the resonant frequency. Fig. 4 shows the two equivalent circuit diagrams. Fig. 4a can be converted to Fig. 4b as follows:

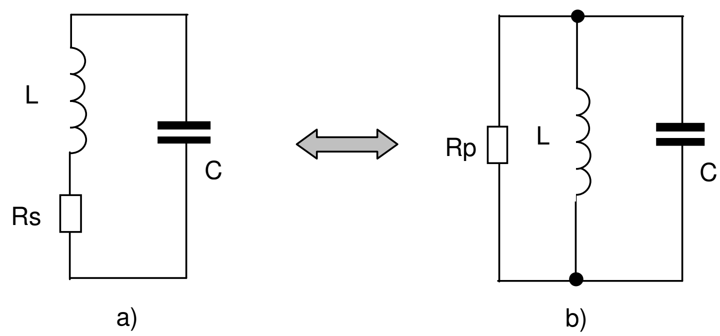

Fig 4. Two equivalent circuits for representing losses in the inductor

$$Q_{F0} = \frac{2\,\pi\,Fo\,L}{Rs}$$

or

$$Q_{F0} = 2\,\pi\,Fo\,C\,Rp$$

where Q_{F0} is the Q of the inductor at a frequency of Fo. L and C are unchanged providing Q_{F0} is more than about 10. Combining these two equations, we arrive at the relationship between Rs and Rp

$$Rp = \frac{L}{C\,Rs}$$

Referring to Fig. 1, C is the sum of C1 and the coupling capacitor C3 – this was called Cnode in Part 2. From Fig. 1, C = 253.4pF and L = 1uH. For Q = 70, Rs is 0.9 Ohms and for Q=200, Rs is 0.314 Ohms. Therefore:-

$$Rp = \frac{3{,}946}{Rs}$$

So, for Q = 70 Rp = 4,384 Ohms and for Q = 200, Rp = 12,567 Ohms.

The required terminating resistance from Fig. 1 is 2,909 Ohms. We can now calculate the revised terminating resistance, using by manipulating the standard resistors-in-parallel formula:

$$\frac{1}{2{,}909} = \frac{1}{4{,}384} + \frac{1}{RT70}$$

where RT70 is the terminating resistance for an inductor with a Q of 70 at the filter centre frequency.

$$RT70 = 8{,}646 \text{ Ohms}$$

and for Q = 200

$$RT200 = 3{,}785 \text{ Ohms}$$

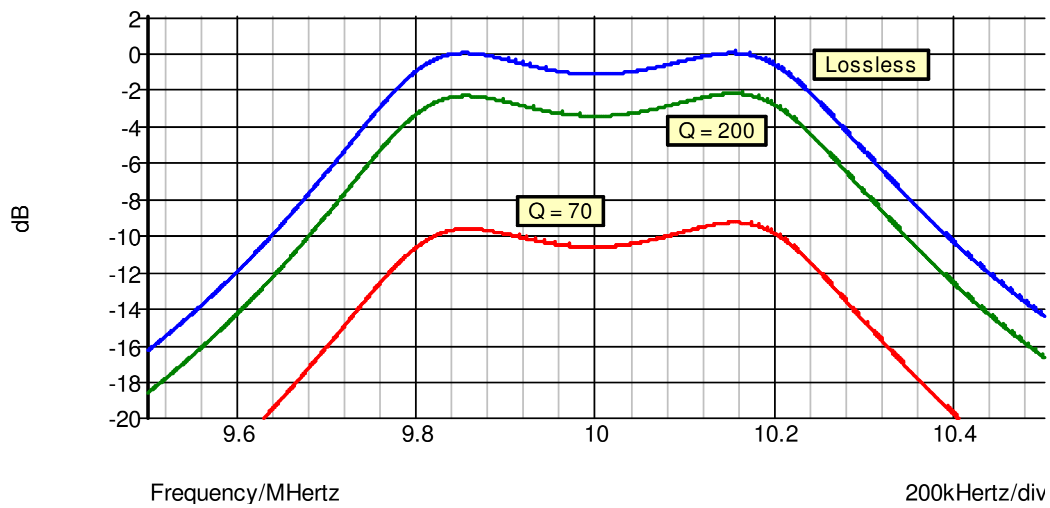

Fig 5. Response with compensated terminating resistors

The shape of the frequency response, when the terminating resistances are adjusted to allow for the loss in the inductors, is unchanged but there is an additional fixed loss at all frequencies. Using a good quality inductor, for example wound onto a toroid iron dust core, will minimise this additional attenuation. Please note that the two-resonator is a special case and it is possible to compensate for the roll-off of the response towards the edges of the pass band as described above. It is not possible achieve complete compensation of the response of circuits with more than two resonators by adjusting the terminating resistors. For many amateur radio applications, two resonators will give adequate performance. Part 8 of this series will look at compensation for the loss in inductors for three or more resonators.

4. Conclusion

The main source of loss within a filter is due to the inductors. The higher circulating currents towards the extremes of the pass band increase the losses and so the pass band becomes more rounded. It is possible to retain the shape of the response, at least for a filter with two resonators, by increasing the values of the terminating resistors but at the expense of an overall increased in attenuation at all frequencies attenuation. Part 8 of this series will examine techniques for compensation for inductor loss in more complex filters.

5. References

1. www.simetrix.co.uk An excellent free fully functioning evaluation version of SPICE.

Richard Harris G3OTK
May 2010
rjharris@iee.org